Test of Comoving Coordinate Frame by Low Magnitude-Redshift Relation

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Abstract

Observational evidence from a variety of sources points at an accelerating universe which is approximately spatially flat and with $(\Omega_M, \Omega_\Lambda)_0 \approx (0.3, 0.7)$. We have shown that for low redshifts, $z \leq 0.2$, the metric of this cosmological model is equivalent to the de Sitter metric within one percent error. Among various coordinate descriptions for the flat de Sitter model two are most widely used, one yields a non-static and RW type, while the other gives a static and Schwarzschild type metric.We have obtained the magnitude-redshift relation in the second coordinate frame. Our result indicates a slope of 2.5. This is in disagreement with observation which is close to the slope of 5. This test discards the second and confirms the first coordinate frame as a comoving frame for matter in de Sitter space. We anticipate this test disqualifies any other solution of the field equation which asymptotically approaches to the static de Sitter metric, e.g. the Schwarzschild-de Sitter metric for space-time around a point mass m.

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As was recently shown by independent groups distant type Ia supernovae provide a striking evidence for an acceleration of the universal expansion quite contrary to the conventional expectations[1-4].A positive cosmological constant is inferred from these measurements[5].

Up to now a satisfactory explanation has not been presented for the existence of a non-zero cosmological constant by the particle physics. Actually, calculations of vacuum energy, which is directly related to the value of cosmological constant, by field theoretical methods give results with a difference of 120 orders of magnitude from the observed values[6]. For these reasons a cosmological constant factor has not been taken seriously in the common discussions of cosmology, but now with these new evidences it seems plausible to consider this factor not to be negligible.

The line element according to FRW model for the de Sitter space, which is appropriate for Λ-dominated worlds, is

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{1}$$

with the scale factor as

$$R(t) = \exp[(\frac{\Lambda}{3})^{1/2} t] \tag{2}$$

where Λ is the cosmological constant.

Today the observatinally favoured universe is near $(\Omega_M, \Omega_\Lambda)_0 \approx (0.3, 0.7)$ and $k = 0$. The Friedmann equation for such a cosmological model may be easily solved and the scalar factor in the FRW metric is given by

$$\frac{R(t)}{R_0} = [\sqrt{\frac{\Omega_{M_0}}{\Omega_{\Lambda_0}}} \sinh \frac{\sqrt{3\Lambda} t}{2}]^{2/3} \tag{3}$$

where R_0 is the scalar factor at our epoch. For $z = 0$ and $z = 0.2$ which correspond to $R(t_0) = R_0$ and $R(t_1) = \frac{R_0}{1.2}$ respectively, equ.(3) gives $\frac{\sqrt{3\Lambda}}{2} t_0 = 1.21$ and $\frac{\sqrt{3\Lambda}}{2} t_1 = 1.99$. It can be shown that

$$\frac{R(t_0)}{R(t_1)} = \{\exp(\sqrt{\frac{\Lambda}{3}}(t_0 - t_1))\}\{1 + 0.01\} \tag{4}$$

Thus in a good approximation we may assume that in this cosmological model the metric for $z \leq 0.2$ is of the de Sitter form. For the rest we restrict our discussion to this range.

For a FRW model, independent of the form of $R(t)$, there is a relation between the distance of a light source and its redshift due to the expansion. This relation for luminosity distance versus redshift expanded as a power series is written as follows,

$$d_L = H_0^{-1}[z + \frac{1}{2}(1 - q_0)z^2 + \ldots], \tag{5}$$

where z is the redshift and H_0 and q_0 are the present values of Hubble's constant and decelaration parameter. Obviously this relation is linear in the first approximation which gives rise to Hubble's law. Also, it can be written in terms of distance modulus,

$$m - M = 25 - 5\log H_0 + 5\log z + 1.086(1-q_0)z + \dots \tag{6}$$

and

$$d_L = 10^{1+(m-M)/5} \tag{7}$$

m and M being apparent and absolute magnitudes of the source, respectively.

On the other hand, there exists an isometry of de Sitter solution that gives a metric of schawrzschild type for which the line element is,

$$ds^2 = (1 - \frac{\Lambda}{3}\rho^2)dT^2 - (1 - \frac{\Lambda}{3}\rho^2)^{-1}d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{8}$$

This solution is static and can be obtained from the ordinary solution by the following transformation,

$$\rho = R(t)r \tag{9}$$

$$T = t - \sqrt{\frac{3}{4\Lambda}}\ln \mid (1 - \frac{\Lambda}{3}\rho^2) \mid \tag{10}$$

The comoving frame is uniquely defined frame of reference in which a given extended set of bodies is at rest or is as much at rest as possible in the sense that their kinetic energy is small as possible. In a cosmological model which satisfies the cosmological principle, a comoving coordinate frame is defined so that the matter distribution has fixed space coordinates. Our aim is to check whether the static de Sitter coordinate frame is eligible to serve as a comoving reference frame. For this purpose by considering the static metric (8) we are going to calculate the redshift - magnitude relation again.

If we suppose that a source is at a point with distance ρ from origin and sends a signal to an observer at the origin, we can find that the gravitational redshift is proportional to the zero-component of metric at ρ [7]. In this case we have

$$g_{00} = (1 - \frac{\Lambda}{3}\rho^2) \tag{11}$$

so that

$$(1+z)^2 = \frac{1}{1 - \frac{\Lambda}{3}\rho^2} \tag{12}$$

and therefore

$$\rho = \sqrt{\frac{3}{\Lambda}\frac{z(2+z)}{(1+z)^2}} \tag{13}$$

On the other hand when a source in the distance ρ emits photons, the energy flux per unit area is diminished by the factor $(1+z)^2$. So for the static metric stated here , we can write the energy flux as follows

$$f = \frac{\mathcal{L}}{4\pi\rho^2(1+z)^2} \tag{14}$$

where $\mathcal{L}$ is the luminosity of source. From (13) and (14) we have

$$f = \frac{\Lambda}{12\pi}\frac{\mathcal{L}}{z(2+z)} \tag{15}$$

In addition , the following relations are used as the definition of relative and absolute magnitudes

$$m = -2.5\log f + k_1 \tag{16}$$

and

$$M = -2.5\log \mathcal{L} + k_2 \tag{17}$$

in which k_1 and k_2 are constants. Now we can write according to (15):

$$m - M = 2.5\log[z(2+z)] - 2.5\log(\frac{\Lambda}{12\pi}) \tag{18}$$

By expanding the function in the brackets in (18) we find the distance modulus-redshift relation in the case of metric (8) as follows:

$$m - M = -2.5\log(\frac{\Lambda}{12\pi}) + 2.5\log z + 1.086(0.69 + \frac{z}{2} - \frac{z^2}{8} + \frac{z^3}{24} - \ldots) \tag{19}$$

A comparison (6) with (19) shows that (6) has a slope of 5 for small z while the slope for (19) is 2.5 . This reveals that the FRW and static de Sitter frames both together do not belong to the same comoving frame of reference. The agreement of observational data by (6) indicates that in the comoving frame of the FRW coordinate frame the galaxies are at rest. We may conclude that this test confirms the FRW coordinates and discards the static de Sitter coordinates as comoving. Thus in the presence of a non-zero cosmological constant the de Sitter form (8) exhibits problems and working in this coordinate as a comoving frame is physically unreasonable.

In fact according to this test, the validity of any solution of the field equation which asymptotically approaches to the static de Sitter metric is under question. The calculations for determining the line element for a point mass m in the presence of Λ are usually carried out in the second coordinate frame i.e. asymptotically approach (8) as a boundary condition. The Schwarzschild-de Sitter metric is

$$ds^2 = (1 - \frac{\Lambda}{3}\rho^2 - \frac{2m}{\rho})dT^2 - (1 - \frac{\Lambda}{3}\rho^2 - \frac{2m}{\rho})^{-1}d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{20}$$

While this solution only maintains spherical symmetry, the physical subspace has homogeneity and isotropy symmetries which shows a disadvantage of (20) in this respect. So if we are going to work in a comoving cosmological frame of reference, we need an alternative coordinate frame in which the metric should asymptotically approach to the non-static de Sitter metric (1), and the isotropy and homogenity of the universe formulated by the cosmological principle is manifested in best way.
This has been done [8], and the result is

$$
\begin{aligned}
ds^2 &= \frac{1}{2}[\sqrt{(1-\frac{2M}{\rho}-\frac{\Lambda}{3}\rho^2)^2+\frac{4\Lambda}{3}\rho^2}+(1-\frac{2M}{\rho}-\frac{\Lambda}{3}\rho^2)]dt^2 \\
&\quad -2e^{2\sqrt{\frac{\Lambda}{3}}t}[[\sqrt{(1-\frac{2M}{\rho}-\frac{\Lambda}{3}\rho^2)^2+\frac{4\Lambda}{3}\rho^2}+(1-\frac{2M}{\rho}-\frac{\Lambda}{3}\rho^2)]^{-1}dr^2 \\
&\quad +\rho^2(d\theta^2+\sin^2\theta\, d\varphi^2)] \\
\rho &\equiv e^{(\sqrt{\frac{\Lambda}{3}}t)}r
\end{aligned}
\tag{21}
$$

On the basis of the above reasoning, eq. (20) is not a coordinate description of a cosmological comoving frame of reference. We should consider (21) as a solution for this system in comoving frame.

It is remarkable that metrics like (8) and (20) could be rejected as acceptable cosmological models according to Weyl's postulate which states that particles of the substratum lie in space-time on a congruence of timelike geodesics diverging from a point in the finite or infinite part. Evidently in (8) and (20) the world lines of comoving observers are not timelike geodesics in the whole space. Our conclusion may be considered as an observational confirmation of this postulate.

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